# PIANO DI ATTIVITA' E PROGETTO DI RICERCA

## PROGETTO DI RICERCA:

Progetto PRIN 2022 '0-Dimensional Schemes, Tensor Theory, and Applications' codice 2022E2Z4AK
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## Descrizione del progetto:

The decomposition of tensors is a procedure which is ubiquitous in many new technologies related with Artificial Intelligence, Algebraic Statistics, Quantum Physics, Network Theory.

The goal of the decomposition process is the separation of elementary components which appear as a mixture in several (biological, chemical, social, linguistic,...) phenomena among the most studied in modern Science. Several branches of Mathematics propose approaches for the study of efficient methods of decompositions, as well as for their theoretical aspects. In recent times researchers realized that deep methods of Algebraic Geometry can be useful to produce sensible advances in the study of the decomposition process. A decomposition can be seen a set of points in a projective space, or in a product of projective spaces, whose images in standard classical maps (Segre maps or Veronese maps) generate the tensor under investigation. It follows that results from the theory of secant varieties, and tools for thedescription of the geometry of finite sets in projective spaces, can provide a fundamental contribution for the improvement of methods of decomposition.

A natural procedure employed in Algebraic Geometry to discover properties of varieties consists of a reduction to limit objects, whose characteristics can be investigated constructively and provide information on general varieties. For finite sets, the degeneration produces 0-dimensional schemes that arise from collision of points. The analysis of general 0-dimensional schemes, e.g. from the point of view of interpolation, is intimately connected with inductive methods for the study of tensors, as well as for direct applications to the decomposition process. Interpolation properties on finite sets in projective spaces have been intensively studied, and their application to symmetric tensors is a recent, but fast developing and well structured theory. Much less is known for properties of 0-dimensional schemes, or properties of finite sets in products of projective spaces. Only initial methods are nowadays available, so that there exists a vast open area of investigation, which promises fruitful applications to tensor theory.

Several methods for the analysis of tensors and their decompositions through the investigation of 0-dimensional schemes have been developed by researchers involved in the project, and appeared in papers published in prestigious journal of pure and applied Mathematics (and one can see in the attached bibliography). The goal of the project is to determine to which extent the new methods can be improved to produce results for the decomposition of symmetric tensors (whose theory is nowadays in great progress), and extended to wider classes of general tensors.

## PIANO DI ATTIVITA'

The winner of the position ("assegnista di ricerca") will work with the Bologna unit of the project on interpolation and resolutions of 0-dimensional schemes, particularly with applications to the study of the dimension of secant varieties for several type of projective varieties. The aim is to develop this kind of studies (with active collaboration with the other units) in order to give more results on varieties which parameterize tensors of several format and properties, thus finding useful ways to represent decompositions of structured tensors.

The unit will mainly address the following topics:

1) Geometric properties of Secant varieties and osculating varieties to varieties relevant for tensor analysis.
2) Interpolation properties of 0-dimensional schemes in general varieties and in projective or multiprojective spaces.
3) Problems on the Geometry of 0-dimensional schemes, related with the decompositions of tensors.